

A New Fracture Function Approach to QCD Initial State Radiation

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Ordinary fracture functions, describing hadrons production in the deep inelastic scattering target fragmentation region, are generalized to account for the production of hadrons in arbitrary number, thus offering a renewed framework for dealing with QCD initial state radiation. We also propose a new jet-like observable which measures beam remnants and low- p_{\perp} scattering fragments and derive its QCD evolution equations by using Jet Calculus. Possible implications for semi-inclusive deep inelastic scattering and hadron-hadron reactions are shortly discussed.

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I. INTRODUCTION

The fracture functions approach [1] allows, within the framework of perturbative Quantum-Chromodynamics, to describe low- p_{\perp} target fragmentation along with semi-hard radiation produced by the active parton participating the hard, hadron-initiated, scattering. Fracture functions are represented as non-perturbative distributions that, once properly generalized to describe multi-particles production, allow an improved description of QCD initial state radiation. In order to specify the connection of those distributions with the standard perturbative QCD approach to the dynamics of initial state, let us consider a fast incoming hadron entering into a high energy reaction. In a QCD-improved parton model approach, the hadron can be represented as a cloud of quasi-real partons which may fluctuate into far off-shell states. If no hard scattering takes place, the virtuality of every single space-like parton would lead the complete cascade to reassemble. If instead a hard scattering occurs, the virtual probe, being able to resolve parton fluctuations in the hadron up to scales of the order $\mathcal{O}(1/Q^2)$, knocks out a parton from the incoming hadron. As a result, the remaining off-shell partons in the initial state cascade cannot reassemble themselves any longer and therefore materialize by starting a partonic initial state shower. This radiation piles up in the final state with the one generated by the struck parton time-like decay in the region of $\mathcal{O}(Q^2)$ transverse momenta. While the latter, due to its detailed knowledge from e^+e^- process, is under better theoretical control, the former is at present still under study, especially in the kinematical regime of present and forthcoming hadron colliders.

In a standard QCD approach to hard processes, initial state radiation is inclusively summed over by using DGLAP evolution equations [2]. The complementary approach in which initial state radiation is unintegrated, or only partially integrated, gives therefore access to a more detailed inspection of space-like parton dynamics at high energy. The aim of this work is thus to introduce multi-particle semi-inclusive fracture functions, which can describe the unintegrated initial state radiation, and to give the corresponding QCD evolution equations.

We use the general theoretical framework of Jet Calculus, originally proposed in Ref. [3] in order to describe jet fragmentation. Within this framework, a highly virtual time-like parton, generated in a hard process, degrades its virtuality emitting a tree-like cascade of coloured quanta of lower mass until the non-perturbative limit is reached and hadronization takes place. Within a leading logarithmic approximation, n-particle cross-sections are given, in the portion of phase space specified by the jet and neglecting for simplicity correlated fragmentations, in terms of single-hadron fragmentation functions. The tree-like structure of the cascade is dictated by leading logarithmic enhancements to the cross-sections given by planar diagrams in the collinear limit. Such enhancements are resummed at all order by DGLAP-like evolution equations [2].

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In the following we apply the same techniques in the space-like case as appropriate for the description of initial state radiation. In order to simplify the notation it is convenient to replace the evolution variable Q^2 with

$$Y = \frac{1}{2\pi\beta_0} \ln \left[\frac{\alpha_s(\mu_R^2)}{\alpha_s(Q^2)} \right], \quad dY = \frac{\alpha_s(Q^2)}{2\pi} \frac{dQ^2}{Q^2}. \quad (1)$$

At LL accuracy the running of α_s is taken into account at one loop by using

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda_{QCD}^2)}, \quad (2)$$

where μ_R^2 is the renormalization scale, Λ_{QCD}^2 is the QCD infrared scale, $\beta_0 = (11C_A - 2n_f)/12\pi$ is the one loop β -function parameter and C_A and n_f are the number of colours and flavours respectively. We then define the variable y

$$y = \frac{1}{2\pi\beta_0} \ln \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right], \quad (3)$$

where Q_0^2 and Q^2 stand for two arbitrary perturbative scales. We next introduce the perturbative parton-to-parton evolution function $E_i^j(x, y)$ which expresses the probability of finding a parton j at the scale Q^2 with a momentum fraction x of the parent parton i at the scale Q_0^2 . The function $E_i^j(x, y)$ satisfies a DGLAP-type evolution equation [3]

$$\frac{\partial}{\partial y} E_i^j(x, y) = \int_x^1 \frac{du}{u} P_k^j(u) E_i^k\left(\frac{x}{u}, y\right), \quad (4)$$

where $P_k^j(u)$ are the space-like splitting functions [2] and a sum over the parton index k is understood. The daughter partons radiated in the evolution process of the active parton, i.e. the virtual cascade of Sec. I, are inclusively summed by eq. (4). The evolution equations can be iteratively solved by using the initial condition

$$E_i^j(x, y)|_{y=0} = \delta_i^j \delta(1-x). \quad (5)$$

The function $E_i^j(x, y)$ resums large collinear logarithms of the type $\alpha_s^n \ln^n(Q^2/Q_0^2)$. Neglecting for simplicity the running of α_s , its expansion at first order in fact reads

$$E_i^j(x, y) \equiv E_i^j(x, Q_0^2, Q^2) \simeq \delta_i^j \delta(1-x) + \frac{\alpha_s}{2\pi} P_i^j(x) \ln \frac{Q^2}{Q_0^2} + \mathcal{O}(\alpha_s^2). \quad (6)$$

The E 's function in LLA therefore well describe the emission of partons that are soft or close to the directions of the emitting parton, while large angle emissions should be included via exact matrix elements. Furthermore, the E 's function satisfies the following *renormalization group* property:

$$E_i^j(x, Q_0^2, Q^2) = \int_x^1 \frac{dw}{w} E_i^k(x/w, Q_0^2, Q_i^2) E_k^j(w, Q_i^2, Q^2), \quad (7)$$

where the scale Q_i^2 is such that $Q_0^2 < Q_i^2 < Q^2$. This property is easily verified once an α_s -expansion is performed on both sides of eq.(7).

II. EVOLUTION EQUATIONS FOR ORDINARY FRACTURE FUNCTIONS

Ordinary fracture functions have been introduced in Ref. [1] in order to give a QCD-based description of semi-inclusive Deep Inelastic Scattering in the target fragmentation region. $M_{h/P}^i(x, z, Q^2)$ represents the conditional probability of finding at a given scale Q^2 a parton i with momentum fraction x of the incoming hadron momentum P while a hadron h with momentum fraction z is detected. All-order factorization of collinear and soft singularities into $M_{h/P}^i(x, z, Q^2)$ were demonstrated in Refs. [4, 5]. In Ref. [6], a fixed order $\mathcal{O}(\alpha_s)$ calculation showed explicitly that the additional collinear singularities occurring when partons are produced in the remnant direction can be properly renormalized only introducing fracture functions. In view of the generalization performed in the next Section, we briefly recall the derivation of the $M_{h/P}^i(x, z, Q^2)$ evolution equation. Let us consider the DIS one-particle inclusive cross-sections $l + P \rightarrow l' + h + X$ in the target fragmentation region, $t = -(P - h)^2 \ll Q^2$,

$$\sigma_T = \int \frac{du}{u} M_{h/P}^i(u, z, Q^2) \hat{\sigma}_i(x/u, Q^2). \quad (8)$$

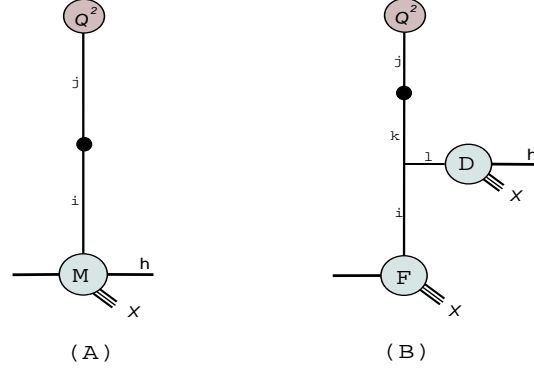


FIG. 1: The A and B term of eq.(9). The black blob denotes the parton-to-parton evolution function E . Partons indices are indicated and at each triple-line vertex is associated a real AP splitting functions $\hat{P}(u)$. The diagrams are at the amplitude square level. The top parton line enters the hard (Q^2) scattering indicated by the bright blob.

The cross-sections is expressed as a convolution of the fracture functions with the point-like partonic cross-sections $\hat{\sigma}^i$. Fixed order calculations shows singularities, as already said, when the emitted parton is collinear to the hadron remnant. The structure of singularities is *rephrased* in a Jet Calculus approach by summing over all combinations of distributions which can give the desired final state. Therefore we may write

$$M_{h/P}^j(x, z, Y) = M_{A,h/P}^j(x, z, Y) + M_{B,h/P}^j(x, z, Y), \quad (9)$$

$$M_{A,h/P}^j(x, z, Y) = \int_x^{1-z} \frac{dw}{w} E_i^j\left(\frac{x}{w}, Y - y_0\right) M_{A,h/P}^i(w, z, y_0), \quad (10)$$

$$M_{B,h/P}^j(x, z, Y) = \int_{y_0}^Y dy \int_{x+z}^1 \frac{dw}{w^2} \int_{\frac{x}{w}}^{1-\frac{z}{w}} \frac{du}{u(1-u)} \cdot E_k^j\left(\frac{x}{wu}, Y - y\right) \hat{P}_i^{kl}(u) D_l^h\left(\frac{z}{w(1-u)}, y\right) F_P^i(w, y). \quad (11)$$

Both terms are depicted in Fig. 1. The variable y_0 corresponds to an arbitrary factorization scale and F_P^i and D_l^h stands respectively for the structure function of a parton i inside a proton, and the fragmentation function of a parton l into the hadron h . $\hat{P}_i^{kl}(u)$ are the real AP splitting functions [3]. The A -term stems for direct hadron production from a fracture function, while the B -term, sometimes called perturbative, takes into account the production of h by a timelike cascade of the daughter parton l . The scale associated to this branching, being not observable, has been integrated over. The integration limits in eq. (9) are fixed by requiring momentum conservation. The evolution equations are easily obtained by deriving eq.(9) with respect to Y . One has

$$\frac{\partial}{\partial Y} M_{A,h/P}^j(x, z, Y) = \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) M_{A,h/P}^j(x/u, z, Y), \quad (12)$$

$$\frac{\partial}{\partial Y} M_{B,h/P}^j(x, z, Y) = \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) M_{B,h/P}^j(x/u, z, Y) + \quad (13)$$

$$\int_x^{\frac{x}{x+z}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}_i^{jl}(u) D_l^h\left(\frac{zu}{x(1-u)}, Y\right) F_P^i(x/u, Y). \quad (14)$$

Restoring the familiar variable Q^2 , we obtain the evolution equation for $M_{h/P}^j(x, z, Q^2)$:

$$Q^2 \frac{\partial}{\partial Q^2} M_{h/P}^j(x, z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) M_{h/P}^i(x/u, z, Q^2) + \quad (15)$$

$$\frac{\alpha_s(Q^2)}{2\pi} \int_x^{\frac{x}{x+z}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}_i^{jl}(u) D_l^h\left(\frac{zu}{x(1-u)}, Q^2\right) F_P^i(x/u, Q^2).$$

$M_{h/P}^j(x, z, Q^2)$ does not depend on the factorization scale and satisfies its own s-channel sum rule [1] :

$$\sum_h \int dz z M_{h/P}^j(x, z, Q^2) = (1-x) F_P^j(x, Q^2). \quad (16)$$

At the phenomenological level, fracture functions have been shown in Ref. [7] to well reproduce at the same time both HERA diffractive and leading proton data, thus convalidating a common perturbative QCD approach to these particular classes of semi-inclusive processes.

III. EVOLUTION EQUATIONS FOR DI-HADRONS FRACTURE FUNCTIONS

Let us consider a double-inclusive Deep Inelastic Scattering process $l + P \rightarrow l' + h_1 + h_2 + X$ where two detected hadrons, h_1 and h_2 , have both $t_{i=1,2} = -(P - h_i)^2 \ll Q^2$. In analogy with eq. (8) we may write the corresponding double-inclusive cross sections as

$$\sigma_T = \int \frac{du}{u} M_{h_1, h_2/P}^j(u, z_1, z_2, Q^2) \hat{\sigma}_j(x/u, Q^2). \quad (17)$$

$M_{h_1, h_2/P}^i(x, z_1, z_2, Q^2)$ gives the conditional probability of finding an active quark i with momentum fraction x of the incoming hadron momentum P while two secondary hadrons with fractional energy z_1 and z_2 are detected. Evolution equations for di-hadron fracture functions $M_{h_1, h_2/P}^i(x, z_1, z_2, Q^2)$ can be obtained generalizing the derivation outlined in the previous Section. $M_{h_1, h_2/P}^i(x, z_1, z_2, Q^2)$ is therefore given by the incoherent sum of all combination of distributions which can give h_1 and h_2 in the final state, see Fig. 2:

$$M_{h_1, h_2/P}^j(x, z_1, z_2, Y) = \sum_{X=A, B, C, D} M_{X, h_1, h_2/P}^j(x, z_1, z_2, Y). \quad (18)$$

Explicitely the four contributions read

$$M_{A, h_1, h_2/P}^j(x, z_1, z_2, Y) = \int_x^{1-z_1-z_2} \frac{dw}{w} E_i^j\left(\frac{x}{w}, Y - y_0\right) M_{A, h_1, h_2/P}^i(w, z_1, z_2, y_0), \quad (19)$$

$$M_{B, h_1, h_2/P}^j(x, z_1, z_2, Y) = \int_{y_0}^Y dy \int_{x+z_1}^{1-z_2} \frac{dw}{w^2} \int_{x/w}^{1-z_1/w} \frac{du}{u(1-u)} E_k^j\left(\frac{x}{wu}, Y - y\right) \hat{P}_i^{kl}(u) \cdot \\ M_{A, h_2/P}^i(w, z_2, y) D_l^{h_1}\left(\frac{z_1}{w(1-u)}, y\right) + (h_1, z_1) \leftrightarrow (h_2, z_2), \quad (20)$$

$$M_{C, h_1, h_2/P}^j(x, z_1, z_2, Y) = \int_{y_0}^Y dy_2 \int_{y_2}^Y dy_1 \int_{x+z_1+z_2}^1 \frac{dw}{w^2} \int_{x+z_1}^{w-z_2} \frac{dx_2}{x_2^2} \int_{x/x_2}^{1-z_1/x_2} \frac{du_1}{u_1(1-u_1)} \hat{P}_m^{nl_1}(u_1) \cdot \\ \int_{x_2/w}^{1-z_2/w} \frac{du_2}{u_2(1-u_2)} \hat{P}_i^{kl_2}(u_2) E_k^m\left(\frac{x_2}{wu_2}, y_1 - y_2\right) E_n^j\left(\frac{x}{u_1 x_2}, Y - y_1\right) \cdot \\ D_{l_2}^{h_2}\left(\frac{z_2}{w(1-u_2)}, y_2\right) D_{l_1}^{h_1}\left(\frac{z_1}{x_2(1-u_1)}, y_1\right) F_P^i(w, y_1) + (h_1, z_1) \leftrightarrow (h_2, z_2), \quad (21)$$

$$M_{D, h_1, h_2/P}^j(x, z_1, z_2, Y) = \int_{y_0}^Y dy \int_{x+z_1+z_2}^1 \frac{dw}{w} \int_{x/w}^{1-(z_1+z_2)/w} \frac{1}{w^2(1-u)^2} \frac{du}{u} E_k^j\left(\frac{x}{wu}, Y - y\right) \cdot \\ \hat{P}_i^{kl}(u) F_P^i(w, y) D_l^{h_1, h_2}\left(\frac{z_1}{w(1-u)}, \frac{z_2}{w(1-u)}, y\right). \quad (22)$$

The A -term is the direct convolution of a di-hadron fracture functions with the function E . The B -term involves one-hadron fracture functions and fragmentation functions. The third and more involved C -term is of a ladder type and contain two y -integration because the two hadrons are emitted by daughters of the active parton ordered by virtualities corresponding to y_1 and y_2 . The D -term involves di-hadron fragmentation functions which obeys its own inhomogeneous-type evolution equations [3, 8]. The B and C terms also contain twin diagrams with h_1 and h_2 interchanged, as indicated by $(h_1, z_1) \leftrightarrow (h_2, z_2)$. All the integration limits in the convolution integrals are fixed by requiring momentum conservation. Taking the Y -derivative of all terms and making repeated use of the evolution equation, eq. (4), and of its intial condition, eq. (5), we obtain

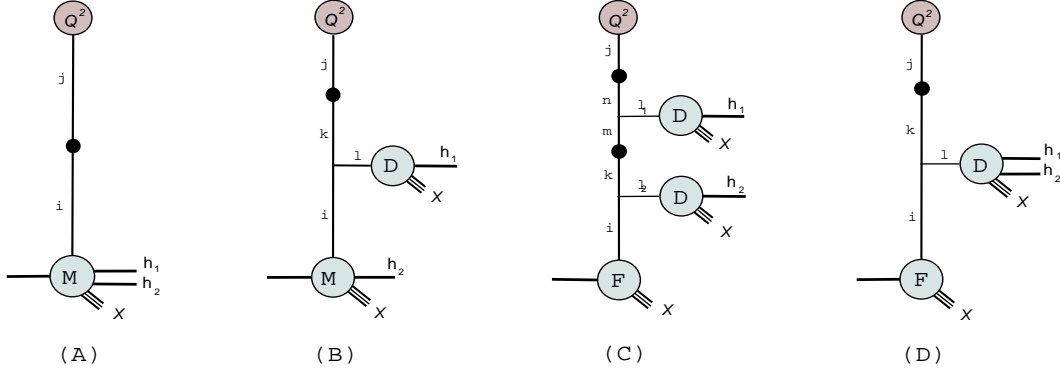


FIG. 2: The four terms of eq. (18). The black blob denotes the parton-to-parton evolution function E . Partons indices are shown and at each triple-line vertex is associated a real AP splitting functions $\hat{P}(u)$. The diagrams are at the amplitude square level. The top parton line enters the hard (Q^2) scattering indicated by the bright blob.

$$\frac{\partial}{\partial Y} M_{A,h_1,h_2/P}^j = \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P_i^j(u) M_{A,h_1,h_2/P}^i(x/u, z_1, z_2, Y) +, \quad (23)$$

$$\begin{aligned} \frac{\partial}{\partial Y} M_{B,h_1,h_2/P}^j &= \int_{\frac{x}{1-z_2}}^{\frac{x}{x+z_1}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}_i^{jl}(u) M_{A,h_2/P}^i(x/u, z_2, Y) D_l^{h_1}\left(\frac{z_1 u}{x(1-u)}, Y\right) + \\ &+ (h_1, z_1) \leftrightarrow (h_2, z_2) + \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P_i^j(u) M_{B,h_1,h_2/P}^i(x/u, z_1, z_2, Y), \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{\partial}{\partial Y} M_{C,h_1,h_2/P}^j &= \int_{\frac{x}{1-z_2}}^{\frac{x}{x+z_1}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}_m^{jl}(u) M_{B,h_2/P}^m(x/u, z_2, Y) D_l^{h_1}\left(\frac{z_1 u}{x(1-u)}, Y\right) \\ &+ (h_1, z_1) \leftrightarrow (h_2, z_2) + \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P_i^j(u) M_{C,h_1,h_2/P}^i(x/u, z_1, z_2, Y), \end{aligned} \quad (25)$$

$$\begin{aligned} \frac{\partial}{\partial Y} M_{D,h_1,h_2/P}^j &= \int_x^{\frac{x}{x+z_1+z_2}} \frac{du}{u} \frac{u^2}{x^2(1-u)^2} \hat{P}_i^{jl}(u) F_P^i(x/u, Y) D_l^{h_1,h_2}\left(\frac{z_1 u}{x(1-u)}, \frac{z_2 u}{x(1-u)}, Y\right) \\ &+ \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P_i^j(u) M_{D,h_1,h_2/P}^i(x/u, z_1, z_2, Y). \end{aligned} \quad (26)$$

Terms containing ordinary splitting functions $P_i^j(u)$ sum up to give a homogeneous term for the evolution equation for $M_{h_1,h_2/P}$. Terms $M_{A,h_2/P}$ and $M_{B,h_2/P}$ in first line of eq. (24) and eq. (25) combine to give $M_{h_2/P}$, eq. (9). We can thus finally write the evolution equation for $M_{h_1,h_2/P}$ as:

$$\begin{aligned} Q^2 \frac{\partial}{\partial Q^2} M_{h_1,h_2/P}^j(x, z_1, z_2, Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P_i^j(u) M_{h_1,h_2/P}^i(x/u, z_1, z_2, Q^2) + \\ &+ \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z_2}}^{\frac{x}{x+z_1}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}_i^{jl}(u) M_{h_2/P}^i(x/u, z_2, Q^2) D_l^{h_1}\left(\frac{z_1 u}{x(1-u)}, Q^2\right) + (h_1, z_1) \leftrightarrow (h_2, z_2) + \\ &+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^{\frac{x}{x+z_1+z_2}} \frac{du}{u} \frac{u^2}{x^2(1-u)^2} \hat{P}_i^{jl}(u) F_P^i(x/u, Q^2) D_l^{h_1,h_2}\left(\frac{z_1 u}{x(1-u)}, \frac{z_2 u}{x(1-u)}, Q^2\right). \end{aligned} \quad (27)$$

As in the case of ordinary fracture functions $M_{h/P}$, eq. (16), also di-hadron fracture functions $M_{h_1,h_2/P}$ can be shown to be independent on the factorization scale variable y_0 and satisfy s-channel sum rules:

$$\sum_{h_2} \int dz_2 z_2 M_{h_1,h_2/P}^i(x, z_1, z_2, Q^2) = (1-x-z_1) M_{h_1/P}^i(x, z_1, Q^2), \quad (28)$$

$$\sum_{h_1,h_2} \int dz_1 z_1 \int dz_2 z_2 M_{2,h_1,h_2/P}^i(x, z_1, z_2, Q^2) = (1-x) F_P^i(x, Q^2). \quad (29)$$

The dihadron fracture function $M_{h_1, h_2/P}$ would be suitable, for example, for studying the coupled strange baryons/mesons production in the target fragmentation region, i.e. $h_1 = \Lambda$ and $h_2 = K$. It should be noted however that fragmentation functions in the strange channel are poorly known at present for one-hadron fragmentation and unknown in the dihadron case.

We wish conclude this sections by working out the general case. Consider a n -inclusive Deep Inelastic Scattering process $l + P \rightarrow l' + h_1 + h_2 + \dots h_n + X$ where the n detected hadrons, h_i , have all $t_i \ll Q^2$, where $t_{i=1, \dots, n} = -(P - h_i)^2$ and fractional energy z_i of the incoming hadron P . In analogy with eq. (8), we may write the corresponding cross sections as

$$\sigma_T = \int \frac{du}{u} M_{h_1, \dots, h_n/P}^j(u, z_1, \dots, z_n, Q^2) \hat{\sigma}_j(x/u, Q^2). \quad (30)$$

Comparing evolution equations for $M_{h/P}$ and $M_{h_1, h_2/P}$, we obtain, by induction, the $M_{h_1, \dots, h_n/P}^j(x, z_1, \dots, z_n, Q^2)$ evolution equation. If $M_{n_1/P}^j$ denotes the n_1 -hadron fracture functions and $D_l^{n_2}$ the n_2 -hadron fragmentation function, such that $n_1 + n_2 = n$, we get

$$\begin{aligned} Q^2 \frac{\partial M_{n/P}^j}{\partial Q^2}(x, z_1, \dots, z_n, Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-\sum_{k=1}^n z_k}}^1 \frac{du}{u} P_i^j(u) M_{n/P}^i(x/u, z_1, \dots, z_n, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \sum_{q=1}^{n-1} \mathcal{P}_n\{h, z\} \cdot \\ &\cdot \int_{\frac{x}{1-\sum_{k=q+1}^n z_k}}^{\frac{x}{1-\sum_{k=1}^q z_k}} \frac{du}{u} \left(\frac{u}{x(1-u)} \right)^q \hat{P}_i^{jl}(u) M_{n-q}^i(x/u, z_{q+1}, \dots, z_n, Q^2) D_l^q\left(\frac{z_1 u}{x(1-u)}, \dots, \frac{z_q u}{x(1-u)}, Q^2 \right) + \\ &+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^{\frac{x}{1-\sum_{k=1}^n z_k}} \frac{du}{u} \left(\frac{u}{x(1-u)} \right)^n \hat{P}_i^{jl}(u) F_P^i(x/u, Q^2) D_l^n\left(\frac{z_1 u}{x(1-u)}, \dots, \frac{z_n u}{x(1-u)}, Q^2 \right). \end{aligned} \quad (31)$$

The inhomogeneous term in the second line of eq. (31) contains a permutation $\mathcal{P}_n\{h, z\}$ over hadrons indices since we have assumed to measure n distinct hadrons. The inner sum takes into account all the possible combinations $M_{n-q} \otimes D_q$ which give the n -hadrons final state configurations. It can be checked that the master formula, eq. (31), reproduces correctly the evolution equation for $M_{h_1, h_2, h_3/P}$ when the latter is explicitly calculated as done for $M_{h_1, h_2/P}$ in the first part of this Section. Fair to say, this is a rather academic exercise. Although the ladder-type kinematics of LL Jet Calculus allows one to write a closed-form evolution equation for $M_{n/P}$, the appearance of an increasing number of unknown distributions prevents any further analysis.

IV. JET APPROACH TO INITIAL STATE RADIATION

The high multiplicity problem can of course be tackled by taking advantage of the some of the properties of QCD radiation. As is well known, hadron activity in a given hard interaction is often collimated in a defined portion of momentum space, this being a signature of the dominant collinear branching of pQCD dynamics. For this reason jet cross-sections are the natural and, possibly, the most effective representation of hadronic final state. This approach avoids the introduction of multi-hadron distributions, which actually has caused the abandon of eq. (31). Perturbative calculations with an arbitrary number of partons in the final state and experimental jet observables can be quantitatively compared only once a common jet-algorithm is chosen and used on both the theoretical and experimental level. Let us sketch the jet approach to initial state radiation described in Ref. [9], in which the k_\perp clustering algorithm is used. The inclusive DIS structure functions F_2 can be decomposed in terms of n -particles exclusive cross-sections $F_2^{(n)}$ as [10]

$$F_2(x, Q^2) = \sum_{n=1}^{\infty} F_2^{(n)}(x, Q^2). \quad (32)$$

If one moves from the exclusive n -particles cross-sections, eq. (32), to the exclusive n -jets cross-sections, a factorized structure emerges [9]

$$F_2^{(n)}(x, Q^2; E_t^2, y_{\text{cut}}) = \sum_{i=q, \bar{q}} \int_x^1 \frac{dz}{z} F_P^i(x/z, \mu_F^2) R_{2,i}^{(n)}\left(z, \alpha_s, \frac{Q^2}{E_t^2}, y_{\text{cut}}\right), \quad (33)$$

where y_{cut} represents the jet resolution parameter and defined in terms of an arbitrary perturbative scale E_t^2 , with $\Lambda^2 \ll E_t^2 \leq Q^2$. In eq. (33), initial state collinear divergences are absorbed into parton distributions functions, F_P^i . The jet-coefficients $R_{2,i}^{(n)}$ are calculable in perturbation theory and, again, are jet-algorithm dependent. The n -jet cross-sections can be calculated by means of fixed order calculations. The main limitation

of such an approach is represented by the technical difficulties of adding more and more partons in the final state. On the contrary, as shown in Ref. [9], a leading logarithmic accurate, Jet Calculus inspired, formulation of n -jet cross-sections is possible. In such an approach, initial state jets, in arbitrary numbers, are accounted for by using a generating functional method [9]. The n -jets cross-sections are then constructed by iterating the block-structure G ,

$$G_i^k(u, Q_i^2, Q_j^2) \equiv \Delta_i^j(Q_i^2, Q_j^2) \hat{P}_j^{lm}(u) J_m(Q_j^2, Q_0^2) \Delta_l^k(Q_l^2, Q_k^2) \quad (34)$$

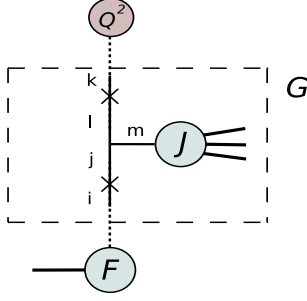


FIG. 3: Representation of one jet-insertion, eq. (34). The crosses represent Sudakov form factors, eq. (35). The three parton lines vertex indicates the real branching controlled by $\hat{P}(u)$.

for each jet insertion and schematically represented in Fig. 3. A Sudakov form factor $\Delta(Q_i^2, Q_j^2)$ [11], defined as

$$\Delta_i^j(Q_i^2, Q_j^2) \equiv \exp \left[- \sum_k \int_{Q_i^2}^{Q_j^2} \frac{dt}{t} \int_{\frac{Q_i^2}{Q_j^2}}^{1 - \frac{Q_i^2}{Q_j^2}} dz \frac{\alpha_s(t)}{2\pi} \hat{P}_i^{jk}(z) \right], \quad (35)$$

inhibits emissions off the struck parton lines between the two scales Q_i^2 and Q_j^2 and thus guarantees that each jet is separated by the nearby jets by a inhibited-emission phase space region. Hard parton emission in eq. (34) is then controlled by real splitting functions $\hat{P}(u)$ and its subsequent time-like evolution is taken into account via the jet function $J(Q^2, k^2)$, defined in Ref. [12], as

$$J(Q^2, k^2) = \int_0^1 dz d(z, Q^2, k^2). \quad (36)$$

In eq. (36), the distribution $d(z, Q^2, k^2)$ expresses the probability that an initial parton, with mass Q^2 , decays into a parton with a longitudinal momentum fraction z with respect to the parent parton with virtual mass $k^2 \ll Q^2$. If the intermediate partons mass k^2 is integrated over we get the parton fragmentation function D ,

$$\int^{Q^2} dk^2 d(z, Q^2, k^2) \equiv D(z, Q^2). \quad (37)$$

In eq. (37), both d and D would have only partonic indices since we are dealing with partonic jets distributions. Such a formulation shares with the Jet Calculus approach the iterative construction. The n -jet cross-sections are built using an alternate *allowed* and *prohibited parton emissions* pattern along the struck parton ordered virtualities, $Q_0^2 < \dots < Q_i^2 < Q_j^2 < Q_k^2 < \dots < Q^2$. The main difference from the Jet Calculus approach resides in the exclusive formulation of eq. (34). The evolution function $E_i^k(u, Q_i^2, Q_k^2)$, in eq. (4), can be regarded however as the analogous, at the inclusive level, of $G_i^k(u, Q_i^2, Q_k^2)$ in eq. (34). The former inclusively resums all parton emissions between the corresponding scales while the latter, instead, constrains such emissions to be approximately collimated in phase space.

V. JET-LIKE FRACTURE FUNCTIONS

The formalism of Ref. [9] indeed can describe jets originating from hard partons decays, whose emission off the active parton can be controlled at the perturbative level. The beam jet is excluded from such a description. Its origin is mainly of soft and kinematical nature since it results from the fragmentation of the spectator partons of the hadron remnants plus, eventually, semi-hard radiation coming from the evolution of the active parton. In the k_\perp -algorithm the beam jet is therefore pre-clustered and not accounted for in the n -jet cross-sections. In view of the importance at present at forthcoming hadron colliders of describing such part of the process, we propose a new semi-inclusive jet-like distribution, $\mathcal{M}_\triangleleft^i(x, Q^2, z, t)$, referring to it as to a jet-like fracture function. $\mathcal{M}_\triangleleft^i(x, Q^2, z, t)$ expresses the probability of finding a parton i with fractional momentum x of the incoming hadron and virtuality Q^2 , while a cluster of hadrons h_i is detected in a portion of phase space \mathcal{R} defined by two variables z and t . The phase space region \mathcal{R} is limited by the constraint

$$\mathcal{R} : t_i = -(P - h_i)^2 < t, \quad t_0 \leq t \leq Q^2. \quad (38)$$

Once the clustering procedure is performed, the variable z is obtained by summing the fractional longitudinal momenta of all hadrons h_i satisfying the phase space constraint, eq. (38):

$$z = \sum_i z_i, \quad h_i \in \mathcal{R}. \quad (39)$$

In analogy with standard inclusive DIS, which makes use of parton distributions functions, we may write

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{dx dQ^2 dz dt} \propto x \sum_{i=q, \bar{q}} e_i^2 \mathcal{M}_\triangleleft^i(x, Q^2, z, t). \quad (40)$$

As in the inclusive case, x and Q^2 are fixed by the scattered lepton kinematics. By defining the n -particle exclusive cross-sections, which may be obtained directly from experiments, as

$$\Sigma_{excl}^{(n)} \equiv \frac{1}{n!} \frac{d^{2n+2} \sigma^{(n)}}{dx dQ^2 \prod_{m=1}^n dz_m dt_m} \quad (41)$$

we may construct the distributions in eq. (40) by implementing the phase space constraints, eq. (38):

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{dx dQ^2 dz dt} \equiv \frac{1}{\sigma_{tot}} \sum_{k=1}^{\infty} \left\{ \prod_{m=1}^k \int_{t_0}^t dt_m \int_0^1 dz_m \right\} \Sigma_{excl}^{(k)} \delta\left(z - \sum_{k=1}^n z_k\right), \quad (42)$$

where t_0 corresponds to beam pipe acceptance loss. At the dynamical level, the active parton described by $\mathcal{M}_\triangleleft^i$ increases its virtuality toward the hard vertex in a series of subsequent branchings. Furthermore, at leading logarithmic accuracy, each emission is strongly ordered, with the softest k_\perp -emissions close to the proton remnant. The invariant momentum transfer constraint in eq. (38) thus perfectly matches this leading logarithmic picture, including soft and semi-hard radiation into $\mathcal{M}_\triangleleft^i$ up to the scale t . All log-enhanced emissions above that scale are then resummed by using the function E , eq. (4). Moreover, at variance with the full inclusive case, the parton initiating the space-like cascade can be specified by inspecting initial state radiation itself. It has a fractional momentum $1 - z$, where z is overall fractional momentum taken away by the hadrons with $t_i \leq t$ and the highest allowed virtuality, t , according to strong ordering. When t is chosen in the perturbative region we may write, according to Jet Calculus,

$$\mathcal{M}_\triangleleft^j(x, Q^2, z, t) = \int_x^{1-z} \frac{dw}{w} \mathcal{M}_\triangleleft^i(w, t, z, t) E_i^j(x/w, t, Q^2). \quad (43)$$

The right hand side of eq. (43) is depicted in Fig. 4. Convolution limits are fixed by requiring momentum conservation. By differentiating eq. (43) with respect to Q^2 , we obtain the evolution equations for $\mathcal{M}_\triangleleft^j$ which reads

$$Q^2 \frac{\partial}{\partial Q^2} \mathcal{M}_\triangleleft^j(x, Q^2, z, t) = \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_k^j(u) \mathcal{M}_\triangleleft^k(x/u, Q^2, z, t). \quad (44)$$

This equation describes how the virtual photon resolves the distributions $\mathcal{M}_\triangleleft^j$, when the virtuality of the latter is varied. As already stated, the evolution equation, eq. (44), actually resums large logarithm of the type $\alpha_s^n \log^n(Q^2/t)$. In real processes, t -ordering is only partially realized. Higher order corrections produce partons that, even if originated by a parent parton with a hard $\hat{t} \geq t$ scale, could give, at the end of the time-like

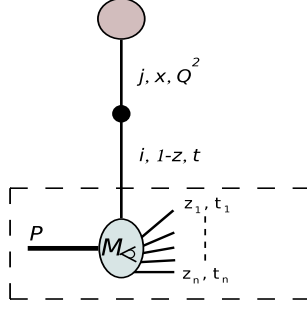


FIG. 4: Representation of the jet-like fracture function $\mathcal{M}_{\triangleleft}^i$ as in eq. (43). The black blob represents the inclusive parton emissions between virtuality t and Q^2 , the bright blob on top the hard scattering process. Also indicated are the space-like parton momentum fractions. The forward radiation off $\mathcal{M}_{\triangleleft}^i$, satisfying the constraint eq. (38), is shown.

shower, final state hadrons with $t_i \leq t$. The clustering procedure of course does not distinguish the origin of such hadrons, nor the virtuality of the parent parton emitted off the space-like chain. All of them are however included in $\mathcal{M}_{\triangleleft}^i$, according to eq. (38). The description of $\mathcal{M}_{\triangleleft}^j$ therefore becomes increasingly reliable as much as the accuracy in space-like partonic shower is enhanced. This, of course, can be achieved by inserting appropriate higher loop splitting functions

$$P_k^j(u) = P_k^{j(0)}(u) + \frac{\alpha_s}{2\pi} P_k^{j(1)}(u) + \dots \quad (45)$$

in the evolution equations, eq. (44). We also note that the latter is formally equivalent to the one for one-particle inclusive extended fracture functions of Ref. [13]. This formal equivalence is expected since we may consider the hadrons contained in \mathcal{R} as a pseudo-particle specified by fractional longitudinal momentum z and invariant momentum transfer t . At the experimental level, the proposed distributions have been already adopted in diffractive DIS measurements at HERA, see Refs. [14, 15]. When the diffractive event is tagged by observing a rapidity gap, the unmeasured low-mass proton excitation and eventually soft p_{\perp} fragments are taken into account by using an integrated distribution as $\mathcal{M}_{\triangleleft}^j$. Of particular interest is the issue concerning the $\mathcal{M}_{\triangleleft}^j$ factorization properties and we will discuss it in the remaining part of this Section. Factorization of ordinary fracture functions were demonstrated in DIS in the single-particle case in Refs. [4, 5]. Since $\mathcal{M}_{\triangleleft}^j$ is actually more inclusive than ordinary extended fracture functions [13], for which factorization holds, we do not expect any factorization breaking effect in DIS. As a result, as for standard parton distribution in inclusive processes, factorization guarantees that once the beam jet and the forward radiation are measured in a given experiment and assigned to $\mathcal{M}_{\triangleleft}^j$, such a distribution can be used in a different experiment solving $\mathcal{M}_{\triangleleft}^j$ evolution equations. Moreover factorization allows a generalization of eq. (33) to include also the beam jet in the n -jet cross-sections, by simply substituting a parton distribution with a jet-like fracture function:

$$F_2^{(n+1)}(x, Q^2; E_t^2, y_{\text{cut}}) = \sum_{i=q, \bar{q}} \int_0^1 dz \int_{\frac{x}{1-z}}^1 \frac{du}{u} \mathcal{M}_{\triangleleft}^i(x/u, Q^2, z, y_{\text{cut}} E_t^2) R_{2,i}^{(n)}\left(u, \alpha_s, \frac{Q^2}{E_t^2}, y_{\text{cut}}\right). \quad (46)$$

In eq. (46), the fraction z of the beam jet, being not measured, is integrated over. The scale t is set to $t \simeq y_{\text{cut}} E_t^2$ and $\mathcal{M}_{\triangleleft}^i$ should be properly evolved according to eq. (44) before being inserted in eq. (46). Turning now to hadron-hadron collisions, consider a semi-inclusive Drell-Yan type process:

$$p + p \rightarrow C_1 + C_2 + \gamma^*_{\rightarrow l+l-} + X. \quad (47)$$

The invariant mass of the lepton-pair provides the perturbative hard scale and the leptonic final state allows a clean inspection of QCD initial state radiation. C_1 and C_2 are two hadronic forward clusters, containing each a beam remnant jet and associated radiation, and are defined in the phase space regions \mathcal{R}_1 and \mathcal{R}_2 , eq. (38), limited by t_1 and t_2 . If we assume for a while that factorization holds, the cross-sections for the process can be written as

$$\frac{d\sigma^{DY}}{dt_1 dt_2 dQ^2 dz_1 dz_2} = \sum_{i,j=q, \bar{q}} \int \int dx_1 dx_2 \left(\mathcal{M}_{\triangleleft}^i(x_1, Q^2, z_1, t_1) \mathcal{M}_{\triangleleft}^j(x_2, Q^2, z_2, t_2) + i \leftrightarrow j \right) \delta(s - x_1 x_2 Q^2), \quad (48)$$

in complete analogy with the inclusive Drell-Yan case, for which instead factorization has been proven in Refs. [16, 17]. Eq. (48), pictorially represented in Fig. 5, does not take into account possible multiple hard interaction, modelled recently in Ref. [18], nor any model for the soft remnant-remnant interaction, which can be instead modelled as in Ref. [19]. Eq. (48) can be used to measure the strenght of factorization breaking effects in semi-inclusive hadron-hadron collisions since factorization properties of the cross sections could be connected with the observed forward radiation pattern.

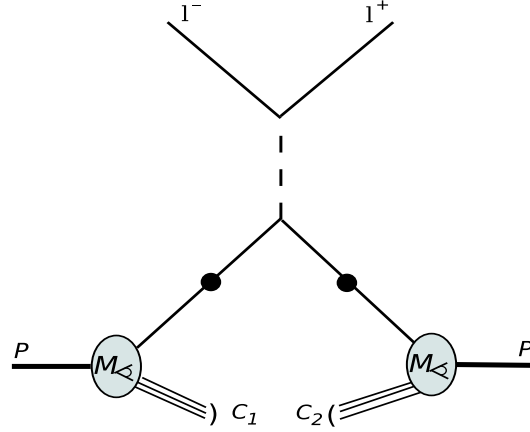


FIG. 5: Representation of a semi-inclusive Drell-Yan process, as in eq. (48). The dashed line indicates the virtual boson.

VI. CONCLUSIONS

In the present work we have developed an alternative QCD-based approach to initial state radiation in hard, hadron-initiated process by using the Jet Calculus and Fracture Function formalism. We propose novel jet-like Fracture Functions which depend explicitly on a minimal set of variables describing forward radiation and target remnants. The lack of knowledge on soft hadronic dynamics compels the use of non-perturbative distributions. As in the inclusive case, the Q^2 -evolution of the proposed semi-inclusive multi-particle distributions can be predicted in QCD. As a result, an explicit description of the QCD initial state radiation becomes possible. In this paper we limit ourselves to results within leading logarithmic approximation. We do not foresee however any serious limitation in implementing higher order corrections and eventually coherence effects in the formalism presented here. Let us add that, since the issue of QCD factorization in hard hadron-hadron reactions is closely connected with the pattern of soft and semi-hard initial state radiation, this novel approach may result in a useful framework to reconsider it. This subject is at the moment under scrutiny as well as further possible applications of the formalism to the dynamics of the minimum bias and of the underlying event at hadron colliders.

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